Calculus Readiness Summer Packet

This packet is designed for students who are preparing for success in Calculus in the upcoming school year.

Name: _____

INTRODUCTION

This "summer packet" answers the questions:

How can I set myself up for success in calculus this year?
How can I avoid feeling lost in class?
What do you expect me to know already?

I have come up with a list of **ten** topics that will help you prepare for success in calculus in the upcoming school year. You can think of each topic as completing the sentence, **"I expect you to already know how to _____."** If you can do these things no-sweat, you will be ready for calculus.

Calculus is, in a sense, a culmination of everything you have learned in math up to this point. That is why I have separated the topics into three categories: **Algebra, Geometry, and Precalculus.** Each topic includes a short note about how it relates to calculus, a list of online resources if you need some help, and 3-10 problems to complete.

While the packet is ***optional*** (unless you were otherwise instructed by either Mr. Beaumont), I recommend working on one topic a week this summer to make sure you are prepared for success in math class next year.

Extra credit will be available for turning this packet in during the first week of school.

PART I: ALGEBRA TOPICS

1. Simplify Complex Fractions
2. Simplify by Factoring
3. Complete the (actual) Square
4. Solve for a Given Variable
5. Find the Inverse

PART II: GEOMETRY TOPICS

6. Write an Equation for a Straight Line Connecting Two Points
7. Problem-Solve with Basic Shapes

PART III: PRECALCULUS TOPICS

8. Use the Unit Circle
9. Find the Domain and Range of a Function
10. Have a General Idea of What Graphs of Functions Look Like

PART I: ALGEBRA

"The hardest part about calculus is the algebra."

- *Everyone*

1. Simplify Complex Fractions

a. We will start off the year discussing something called a *limit.* It is the fundamental concept behind calculus, and it is a weird concept because it is unlike anything you have learned in math up to this point. However, I find that students don't really struggle with the concept of a limit; they struggle with the algebra associated with solving limit problems. The algebra can get dense, but if you can handle adding, subtracting, multiplying, dividing, and simplifying fractions—even the hard ones—and applying *Exponent Rules* with fractions, you will be good to go.

b. Helpful resources if you feel like you need a refresher on how to do this:
   i. Video: *Advanced Algebra - Simplify Complex Rational Fractions* https://youtu.be/ZNibz7WAU0g
   ii. **Video: *Dividing Fractions - Why invert and multiply?*** 🙌 🙌 https://youtu.be/-sHrrChSRq0
   iii. Video: *Exponent Rules with Examples* https://youtu.be/b4mSqcJND3I
   iv. **Online Practice: *Simplify Expressions, More Complicated-Section*** https://www.geogebra.org/m/YjhAuyu4
   v. Online Practice: *Simplifying Radicals* https://www.geogebra.org/m/nufHUrek
   vi. An ***awesome*** visual for multiplying fractions: https://www.geogebra.org/m/AZnX7deX
   vii. An interesting visual for dividing fractions: https://www.geogebra.org/m/k5X8GnJa

c. Complete these practice problems.
   i. Simplify the expression by getting rid of the fraction and combining like-terms $\frac{a(2/b)}{(3/a)}$
   ii. Simplify $\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

Write each of the following expressions in the form $ca^p b^q$ *where* c, p, *and* a *are numbers (in other words, get rid of all of the fractions and radicals and then combine like-terms).*

   iii. $\frac{y^2\sqrt{xy}}{y^2x^4}$
   iv. $\frac{(2a)^3}{b}$
   v. $\frac{a(2/b)}{(b^{-1})\sqrt{a}}$
   vi. $\left(\frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2\left(\frac{b^{\frac{2}{3}}}{a^{\frac{1}{2}}}\right)$

2. **Simplify by Factoring**

   a. As I mentioned earlier, the algebra related to our first topics (*limits*) can be dense. We will oftentimes try to simplify the algebra on our end of these problems by *factoring* either the numerator or denominator of a fraction to get something to cancel out. Examples (iii)-(v) below are fantastic examples of what I am talking about.

   b. Helpful resources if you feel like you need a refresher on how to do this:
      i. Video: *Simplifying by Factoring* https://youtu.be/SRudSszA1l0
      ii. **Online practice with several levels of difficulty: *Endless Factoring Practice* https://www.geogebra.org/m/MzaJ3Tvg**

   c. Complete these practice problems.
      Factor completely.
      i. $x^2 + 2x - 3$
      ii. $x^6 - 16x^4$

      Simplify the following expressions by factoring.
      iii. $\frac{ab-a}{b^2-b}$
      iv. $\frac{x^3-9x}{x^2-7x+12}$
      v. $\frac{x^2-2x-8}{x^3+x^2-2x}$

3. **Complete the (actual) Square**

   a. Sometimes it will be necessary to complete the square to get functions into a form in which we can either graph them, take their *limit*, calculate their *derivative*, or find their *integral* more easily. This year we discussed algebraic and visual ways to "complete the square," which will come in handy in calculus—don't forget it all over the summer!

   b. Helpful resources if you feel like you need a refresher on how to do this:
      i. **Video: *Beautiful Visual Explanation of Completing the Square*** 🙌 🙌 **https://youtu.be/McDdEw_Fb5E**
      ii. Video: Example problem https://youtu.be/bNQY0z76M5A
      iii. Interactive Activity: a visual for completing the square https://www.geogebra.org/m/bSkVv9nd
      iv. Interactive Activity: step-by-step how to complete the square https://www.geogebra.org/m/Zj2AP2GA
      v. Online practice: https://www.geogebra.org/m/ZuefKJW6

   c. Complete these practice problems.
      For each equation, complete the square and reduce to one of the standard forms $y - y_1 = A(x - x_1)^2$ *or* $x - x_1 = A(y - y_1)^2$
      i. $y = x^2 + 4x + 3$
      ii. $3x^2 + 3x + 2y = 0$
      iii. $9y^2 - 6y - 9 - x = 9$

4. **Solve for a Given Variable**

   a. Calculus is sometimes called "the mathematics of change." We want to know how changes in things like time affect changes in population growth, how changes in population density affect changes in the spread of viruses. In order to be able to study how a change in one variable causes or correlates with a change in another variable, you need to know how to solve for a given variable.

   b. Helpful resources if you feel like you need a refresher on how to do this:
      i. **Video: *Algebra Changing the Subject of an Equation (rearranging formulae)*** 🙌 **https://youtu.be/4rBR6DUpQkQ**
      ii. Video: *Solving variables on both sides - advanced* https://youtu.be/FUeuPC0ONC8

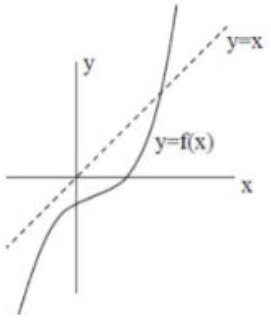
   c. Complete these practice problems.
      i. Solve for x: $x^6 - 16x^4 = 0$
      ii. Solve for x: $2x + 1 = \frac{5}{x+2}$
      iii. Solve for x: $\frac{x+1}{x} - \frac{x}{x+1} = 0$
      iv. Solve for a: $V = 2(ab + bc + ca)$
      v. Solve for a: $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
      vi. Solve for d: $2x - 2yd = y + xd$

5. **Find the Inverse**

a. Finding the inverse of a function goes hand-in-hand with solving for a given variable. Sometimes solving for a given variable *requires* applying the inverse function to both sides of an equation, as is the case with many exponential and logarithmic equations. We may also want to analyze the inverse function of some relationship we are studying. For example, we may want to compare how changes in a wolf population correspond with changes in a rabbit population *and vice versa.* I expect you to be able to find inverse functions, sketch them on a graph, and use them to solve for a given variable.

b. Helpful resources if you feel like you need a refresher on how to do this:
   i. Video: *How To Find The Inverse of a Function* https://youtu.be/2zeYEx4eTdc
   ii. Interactive Online Activity: *Graphs of functions and their inverses* https://www.geogebra.org/m/S9mgw5vv
   iii. Video: *Solving exponential equations using logarithms: base-2* https://youtu.be/7Ig6kVZaWoU
   iv. Video: *Logarithmic equations: variable in the argument* https://youtu.be/Kv2iHde7Xgw
   v. Video: *Vi Hart's monologue on logarithms* https://youtu.be/N-7tcTIrers

c. Complete these practice problems.
   i. A function $f(x)$ has the graph below. Sketch the graph of its inverse $f^{-1}(x)$



Find the inverse of the following functions.

   ii. $f(x) = 2x + 3$
   iii. $f(x) = \frac{x+2}{5x-1}$
   iv. $f(x) = x^2 - 2x - 1,\ x > 0$

Solve for x using the inverses of exponential and logarithmic functions

   v. $5^{(x+1)} = 25$
   vi. $\frac{1}{3} = 3^{(2x+2)}$
   vii. $log_2(x) = 3$

PART II: GEOMETRY

"You'll never use geometry."

- *Liars*

6. **Write an Equation for a Straight Line**

An example of what I will say in class	**What I will *not* say in class**
Write an equation for a straight line that passes between $(-2, 7)$ and $(2, 3)$.	Let's consider the equation for a straight line that passes between $(-2, 7)$ and $(2, 3)$. We know that the *standard equation* for a line has the form $y - y_1 = m(x - x_1)$ The slope m is *the change in y over the change in x* between these two points, given by: $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-7}{2-(-2)} = \frac{-4}{4} = -1$ Now, we can plug in the slope m and substitute x_1 and y_1 with the x and y values of one of our points so that we get: $y + 2 = -1(x - 7)$ We can solve for y and distribute m to write this equation in point-slope form-- that is $y = mx + b$. When we do this, we get $y = -x + 5$

a. <u>You absolutely must know how to write an equation for a straight line.</u> You must. You must. **You must.** I also expect you to know how to find equations for *parallel* and *perpendicular* lines for a given line.
b. Helpful resources if you feel like you need a refresher on how to do this:
    i. Video: *How to find the equation of a line given two points* https://youtu.be/4vXqMsvPSv4
    ii. A site where you can practice writing the equation for a line in standard form and point-slope form and then check your work: *Geogebra* https://www.geogebra.org/m/Gs8JFJKH
    iii. Video: *Equations of parallel and perpendicular lines* https://youtu.be/9hryH94KFJA
    iv. A visual for parallel and perpendicular lines https://www.geogebra.org/m/n6NA6f9J
c. Complete these practice problems.
    i. Write the equation of the line through $(-1, 3)$ and $(2, -4)$.
    ii. Write the equation of the line through $(-1, 4)$ and $(3, 2)$ in point-slope form.
    iii. Determine the equation of the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$.
    iv. Shade the region in the xy-plane that is described the following inequalities

$$3x - y - 7 < 0$$
$$x + 5y + 3 \geq 0$$

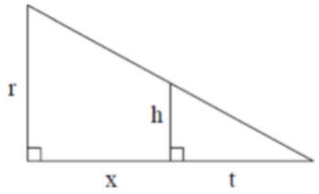
7. **Problem-Solve with Basic Shapes**

a. These problems not only help you develop creative problem-solving skills, they also provide great examples for studying the *relationships* between different variables.

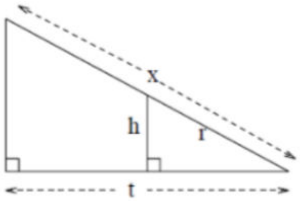
b. Helpful resources if you feel like you need a refresher on how to do this:
   i. How to solve a problem similar to #7***ci*** below: https://youtu.be/1T9dHMU5pkk
   ii. How to solve a problem similar to #7***cii*** below: https://youtu.be/qYae-k8Kcsg
   iii. How to solve a problem similar to #7***ciii*** below: https://youtu.be/DusixDqRgwQ
   iv. How to solve a problem similar to #7***civ*** below: https://youtu.be/DbQAlHSOLUo

c. Complete these practice problems.

   i. Express *x* in terms of the other variables in the picture below.



   ii. Express *x* in terms of the other variables in the picture below.



   iii. Find a formula for the *perimeter* of a window of the shape in the picture shown below.

   iv. Find the ratio of the area in the shaded region to the area of the square in the circle below.

PART III: PRECALCULUS

"Refuse to Lose (what you learned in precal)"

-Nike (and Mr. Beaumont)

8. **Use the Unit Circle**

   a. We spent a lot of time studying the Unit Circle this year, and that is because the key points on the Unit Circle are like the ABC's of trigonometric and sinusoidal functions. If you have taken physics already, you know that *many* systems and real-world phenomena can be described with sinusoidal functions, and we will study some of them in calculus.

      We will be using the Unit Circle like a *periodic table*. You don't need to memorize it, but you need to know how to find values of sine, cosi ne, tangent, etc. at various angles.

   b. Helpful resources if you feel like you need a refresher on how to do this:
      i. Ms. Leece's Unit Circle: https://docs.google.com/document/d/1GavTF1cTIybSsrWRawqFFpTGzwe4LHrserYntWkWJYw/edit?usp=sharing
      ii. Someone else's Unit Circle: http://www.embeddedmath.com/downloads/index.php?item=unitcircle.php
      iii. Blank Unit Circle: https://docs.google.com/document/d/1HwGuBYUfekERIVsaxRL4v4RX7FciBsqdNoPBg9JAKzU/edit?usp=sharing
      iv. Video: *Master the Unit Circle!* https://youtu.be/iHa8f7k4Tg8
      v. Interactive Online Resource: *Geogebra* https://www.geogebra.org/m/G7xgNRxm

   c. Complete these practice problems.
      i. $cos(210^o)$
      ii. $sin(\frac{5\pi}{4})$
      iii. $tan^{-1}(1)$
      iv. $sin^{-1}(1)$
      v. $cos(\frac{9\pi}{4})$
      vi. $sin^{-1}(\frac{\sqrt{3}}{2})$
      vii. $tan(\frac{7\pi}{6})$

9. **Find the Domain and Range of a Function**

   a. To study functions, you need to be able to identify their domain (extent along the *x*-axis) and range (extent along the *y*-axis). We spent a lot of time discussing when a function is *undefined* and how that affects the domain and how to practically find the domain and range of a function. This will be very important in calculus, and I expect you to know how to do it.

   b. Helpful resources if you feel like you need a refresher on how to do this:
      i. Video: *What is the domain of a function?* https://youtu.be/-DTMakGDZAw
      ii. Video: *How to find the domain of a rational function - domain and range* https://youtu.be/5lYSYUmZZ78
      iii. Video: *How to find the implied domain of a function* https://youtu.be/C1Cssyc3t24
      iv. Graphing calculator: *desmos.com*
      v. *Domain and Range Animation* https://www.geogebra.org/m/FCMDfPNf
      vi. Online Interactive Activity: *Domain and range of real functions* https://www.geogebra.org/m/Ggyghyny

   c. Complete these practice problems.

      Find the domain and range of the following functions. Feel free to use the graphing tool at desmos.com to answer these questions.

      i. $f(x) = 7$
      ii. $g(x) = \frac{5x-3}{2x+1}$
      iii. $f(x) = \frac{|x|}{x}$
      iv. $g(x) = \sqrt{x - 2}$
      v. $f(x) = \frac{3x+1}{\sqrt{x^2+x-2}}$

10. **Have General Idea of What Graphs of Functions Look Like**

a. Last, but certainly not least—actually probably most important (next to knowing how to write an equation for a straight line)—is having a general idea of what graphs of functions look like. This is what precal is all about and what precal should have prepared you to do. We will talk about polynomial, exponential, logarithmic, sinusoidal, trigonometric, absolute value, rational, and radical functions, *and you should have a general idea of what these curves look like.* For example, you should be able to hear what type of function we are dealing with and know:
   - If it is curvy, flat, or steep
   - What happens to the function as *x* get really big in the positive direction or negative direction
   - If it is undefined anywhere
   - If its range is limited
   - What kind of symmetry it has

   You should also be able to recognize *transformations* of functions in an equation and know how that relates to the graph. For example, you should remember that the transformation $f(x-3)$ shifts the function $f(x)$ to the right by 3 and $-f(x)$ flips the function $f(x)$ about the *x*-axis.

b. Helpful resources if you feel like you need a refresher on how to do this:
   - **i. Desmos: *Card Sort: Transformations***
     https://student.desmos.com?prepopulateCode=jaf333
   - ii. Desmos: *Match my Parabola*
     https://student.desmos.com?prepopulateCode=ef7p6m
   - iii. Desmos: *Card Sort: Exponentials*
     https://student.desmos.com?prepopulateCode=kmbuk8
   - iv. Interactive Activity: *Functions transformations-square root, quadratic, abs value*
     https://www.geogebra.org/m/GMj893Jq
   - **v. Parent Functions Charts:**
     https://people.clas.ufl.edu/srnatkins/files/ParentFunctionChart.pdf
   - vi. Video: *The Parent Function Graphs and Transformations*
     https://youtu.be/8howrlJVyxw

c. Complete these practice problems.
   i. Match the graphs with their descriptions at the link below.
   https://student.desmos.com?prepopulateCode=cah25j

The graph of the function $y = f(x)$ is given below.
Determine the graphs of the following functions:

ii. $f(x+1)$

iii. $f(-x)$

iv. $|f(x)|$